M.F

SEMINAR 4

APLICAȚII LA CINEMATICA FLUIDELOR

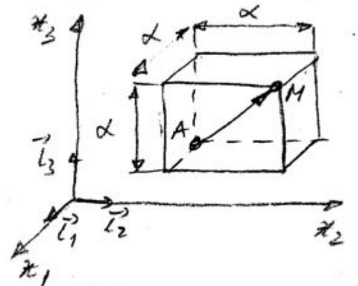
① În raport cu un reper cartezian, viteza unui fluid este:

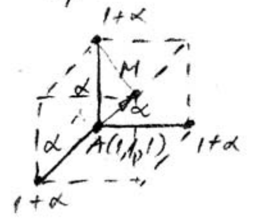
$$\vec{v} = x_1^2 x_2 \vec{i}_1 + x_1 x_2^2 \vec{i}_2 + x_2 x_3 \vec{i}_3$$

În punctul $A(1,1,1)$ ca vârf, se consideră un cub cu muchia α. Se cere să se determine viteza în vârful opus M al cubului, în două variante:

a) folosind relația: $\vec{v} = v_1 \vec{i}_1 + v_2 \vec{i}_2 + v_3 \vec{i}_3$

b) folosind relația Cauchy - Helmholtz: $\vec{v}_M = \vec{v}_A + \vec{\omega}_A \times \vec{AM} + \nabla\phi$





Rezolvare

a) $v_1 = x_1^2 x_2$; $v_2 = x_1 x_2^2$; $v_3 = x_2 x_3$

$M(1+\alpha,\ 1+\alpha,\ 1+\alpha)$

$$\vec{v}_M = (1+\alpha)^2(1+\alpha)\vec{i}_1 + (1+\alpha)(1+\alpha)^2\vec{i}_2 + (1+\alpha)(1+\alpha)$$

$$\boxed{\vec{v}_M = (1+\alpha)^3\vec{i}_1 + (1+\alpha)^3\vec{i}_2 + (1+\alpha)^2\vec{i}_3}$$

b) $\vec{v}_A = 1^2 \cdot 1\,\vec{i}_1 + 1\cdot 1^2\vec{i}_2 + 1\cdot 1\,\vec{i}_3 = \vec{i}_1 + \vec{i}_2 + \vec{i}_3$

$$\vec{\omega}_A = \frac{1}{2}\,\mathrm{rot}\,\vec{v}_A = \frac{1}{2}\begin{vmatrix} \vec{i}_1 & \vec{i}_2 & \vec{i}_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ x_1^2 x_2 & x_1 x_2^2 & x_2 x_3 \end{vmatrix} =$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial x_2}(x_2 x_3)\vec{i}_1 + \frac{\partial}{\partial x_1}(x_1 x_2^2)\vec{i}_3 + \frac{\partial}{\partial x_3}(x_1^2 x_2)\vec{i}_2 - \frac{\partial}{\partial x_2}(x_1^2 x_2)\vec{i}_3 - \right.$$

$$\left. - \frac{\partial}{\partial x_3}(x_1 x_2^2)\vec{i}_1 - \frac{\partial}{\partial x_1}(x_2 x_3)\vec{i}_2\right] = \frac{1}{2}\left(x_3\vec{i}_1 + x_2^2\vec{i}_3 + 0 - \right.$$

$$\left. - x_1^2\vec{i}_3 - 0 - 0\right) = \frac{1}{2}\cdot 1\,\vec{i}_1 + \cancel{\frac{1}{2}\cdot 1\,\vec{i}_3} - \cancel{\frac{1}{2}\cdot 1\,\vec{i}_3} = \frac{1}{2}\vec{i}_1$$

$$\vec{AM} = \alpha\vec{i}_1 + \alpha\vec{i}_2 + \alpha\vec{i}_3$$

$$\vec{\omega}\times\vec{AM} = \begin{vmatrix} \vec{i}_1 & \vec{i}_2 & \vec{i}_3 \\ \frac{1}{2} & 0 & 0 \\ \alpha & \alpha & \alpha \end{vmatrix} = 0\cdot\alpha\vec{i}_1 + \frac{1}{2}\cdot\alpha\vec{i}_3 + 0\cdot\alpha\vec{i}_2 -$$

$$- 0\cdot\alpha\vec{i}_3 - 0\cdot\alpha\vec{i}_1 - \frac{1}{2}\cdot\alpha\vec{i}_2 = \frac{\alpha}{2}(\vec{i}_3 - \vec{i}_2)$$

$$\phi_A = a_{11}X_1^2 + a_{22}X_2^2 + a_{33}X_3^2 + 2(a_{12}X_1X_2 + a_{13}X_1X_3 + a_{23}X_2X_3)$$

$$X_1 = X_2 = X_3 = \alpha$$

$$\nabla\phi_A = \frac{\partial\phi_A}{\partial x_1}\vec{i}_1 + \frac{\partial\phi_A}{\partial x_2}\vec{i}_2 + \frac{\partial\phi_A}{\partial x_3}\vec{i}_3 =$$

$$= (a_{11}^A X_1 + a_{12}^A X_2 + a_{13}^A X_3)\vec{i}_1 + (a_{21}^A X_1 + a_{22}^A X_2 + a_{23}^A X_3)\vec{i}_2 +$$

$$+ (a_{31}^A X_1 + a_{32}^A X_2 + a_{33} X_3)\vec{i}_3$$

$$a_{11}^A = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_1}\right) = \frac{1}{2}\cdot 2\frac{\partial v_1}{\partial x_1} = \frac{\partial(x_1^2 x_2)}{\partial x_1} = 2x_1x_2\Big|_A = 2$$

$$a_{22}^A = \frac{1}{2}\left(\frac{\partial v_2}{\partial x_2} + \frac{\partial v_2}{\partial x_2}\right) = \frac{1}{2}\cdot 2\frac{\partial v_2}{\partial x_2} = \frac{\partial(x_1 x_2^2)}{\partial x_2} = 2x_1x_2\Big|_A = 2$$

$$a_{33}^A = \frac{1}{2}\left(\frac{\partial v_3}{\partial x_3} + \frac{\partial v_3}{\partial x_3}\right) = \frac{1}{2}\cdot 2\cdot\frac{\partial(x_2x_3)}{\partial x_3} = x_2\Big|_A = 1$$

$$a_{12}^A = a_{21}^A = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) = \frac{1}{2}\left(\frac{\partial(x_1^2x_2)}{\partial x_2} + \frac{\partial(x_1x_2^2)}{x_1}\right) = \frac{1}{2}(x_1^2 + x_2^2)\Big|_A = 1$$

$$a_{13}^A = a_{31}^A = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_3} + \frac{\partial v_3}{\partial x_1}\right) = \frac{1}{2}\left(\frac{\partial(x_1^2x_2)}{x_3} + \frac{\partial(x_2x_3)}{\partial x_1}\right) = 0$$

$$a_{23}^A = a_{32}^A = \frac{1}{2}\left(\frac{\partial v_2}{\partial x_3} + \frac{\partial v_3}{\partial x_2}\right) = \frac{1}{2}\left(\frac{\partial(x_1x_2^2)}{\partial x_3} + \frac{\partial(x_2x_3)}{\partial x_2}\right) = \frac{1}{2}(0 + x_3)\Big|_A = \frac{1}{2}$$

$$\nabla\phi_A = (2\alpha + \alpha + 0)\vec{i}_1 + (\alpha + 2\alpha + 0{,}5\alpha)\vec{i}_2 + (0 + 0{,}5\alpha + \alpha)\vec{i}_3 =$$

$$= 3\alpha\vec{i}_1 + 3{,}5\alpha\vec{i}_2 + 1{,}5\alpha\vec{i}_3$$

$$\vec{v}_M = \vec{i}_1 + \vec{i}_2 + \vec{i}_3 + 0{,}5\alpha\vec{i}_3 - 0{,}5\alpha\vec{i}_2 + 3\alpha\vec{i}_1 + 3{,}5\alpha\vec{i}_2 + 1{,}5\alpha\vec{i}_3 =$$

$$= (1+3\alpha)\vec{i}_1 + (1 - 0{,}5\alpha + 3{,}5\alpha)\vec{i}_2 + (1 + 0{,}5\alpha + 1{,}5\alpha)\vec{i}_3$$

$$\boxed{\vec{v}_M = (1+3\alpha)\vec{i}_1 + (1+3\alpha)\vec{i}_2 + (1+2\alpha)\vec{i}_3}$$

2. Câmpul de viteze al unei mişcări permanente este dat de vitezele: $v_1 = 3x_2$; $v_2 = 2x_1$

Să se determine viteza şi acceleraţia în punctul $A(x_1 = 3, x_2 = 5)$ şi să se scrie ecuaţia liniei de curent care trece prin acest punct.

Rezolvare:

$$\vec{v}_A = v_{1A}\vec{i}_1 + v_{2A}\vec{i}_2 + v_{3A}\vec{i}_3 = 3x_2\vec{i}_1 + 2x_1\vec{i}_2 + 0\vec{i}_3 =$$

$$= 3\cdot 5\vec{i}_1 + 2\cdot 3\vec{i}_2 + 0\cdot\vec{i}_3 = 15\vec{i}_1 + 6\vec{i}_2$$

În normă:

$$\|\vec{v}_A\| = \sqrt{15^2+6^2} = 16,15\ m/s$$

$$\vec{a}_A = a_1\vec{i}_1 + a_2\vec{i}_2 + a_3\vec{i}_3$$

$$a_1^A = v_1\frac{\partial v_1}{\partial x_1} + v_2\frac{\partial v_1}{\partial x_2} + v_3\frac{\partial v_1}{\partial x_3} = 0 + 2x_1\cdot 3 + 0 = 18$$

$$a_2^A = v_1\frac{\partial v_2}{\partial x_1} + v_2\frac{\partial v_2}{\partial x_2} + v_3\frac{\partial v_3}{\partial x_3} = 3x_2\cdot 2 + 0 + 0 = 30$$

$$a_3^A = v_1\frac{\partial v_3}{\partial x_1} + v_2\frac{\partial v_3}{\partial x_2} + v_3\frac{\partial v_3}{\partial x_3} = 0$$

$$\vec{a}_A = 18\vec{i}_1 + 30\vec{i}_2$$

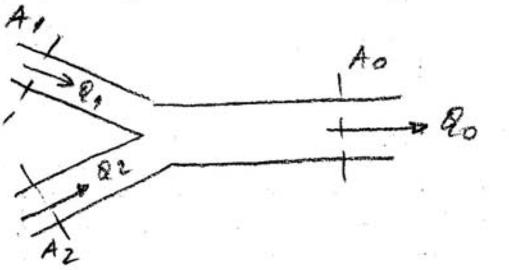
$$\|\vec{a}_A\| = \sqrt{18^2+30^2} = 34,98\ m/s^2$$

Ec. liniei de curent care trece prin A:

$$\frac{dx_1}{v_1(x_1,x_2,x_3)} = \frac{dx_2}{v_2(x_1,x_2,x_3)} = \frac{dx_3}{v_3(x_1,x_2,x_3)} \Rightarrow \frac{dx_1}{3x_2} = \frac{dx_2}{2x_1}$$

$$\Rightarrow 2x_1\,dx_1 = 3x_2\,dx_2$$

(3) Admițând aceeași viteză medie în toate cele trei ramificații din figura de mai jos, să se determine aria A_2 și debitele Q_1 și Q_0, cunoscându-se $A_0 = 0,8m^2$; $A_1 = 0,45m^2$, $Q_2 = 80\ l/s$.



Rezolvare:

Se aplică ecuația de continuitate:

$$Q_1 + Q_2 = Q_0$$

sau:

$$v_1\cdot A_1 + Q_2 = v_0\cdot A_0.$$

Dar: $v_1 = v_0 = v_m$ (viteza medie)

$$v_m A_1 + Q_2 = v_m A_0$$

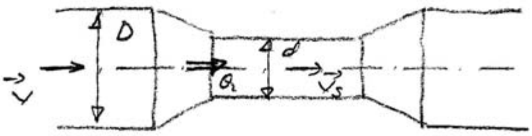
$$Q_2 = v_m(A_0 - A_1) \Rightarrow v_m = \frac{Q_2}{A_0 - A_1} = \frac{80\cdot 10^{-3}}{0,8-0,45} = 0,22\ m/s$$

$$Q_1 = v_m\cdot A_1 = 0,22\cdot 0,45 = 0,1\ m^3/s$$

$$Q_0 = v_m\cdot A_0 = 0,22\cdot 0,8 = 0,175\ m^3/s$$

$$A_2 = \frac{Q_2}{v_m} = \frac{80\cdot 10^{-3}}{0,22} \simeq 0,35\ m^2$$

(4) O conductă de diametru $D = 200$ mm are o zonă de strangulare de diametru $d = 30$ mm. Prin conductă curge un debit $Q = 30$ l/s de petrol ($\rho_p = 810$ kg/m^3). Să se determine debitul masic Q_M, viteza medie v în conductă și viteza v_s în secțiunea strangulată.



Rezolvare

$$Q = v \cdot \frac{\pi D^2}{4} = v_s \cdot \frac{\pi d^2}{4}$$

$$\Rightarrow v = \frac{4Q}{\pi D^2} = \frac{4 \cdot 30 \cdot 10^{-3}}{\pi \cdot 0{,}2^2} = 0{,}954 \text{ m/s}$$

$$v_s = \frac{4Q}{\pi d^2} = \frac{4 \cdot 30 \cdot 10^{-3}}{\pi \cdot 0{,}03^2} = 42{,}44 \text{ m/s}$$

$$Q_M = \rho \cdot Q = 810 \cdot 30 \cdot 10^{-3} = 24{,}3 \text{ kg/s} \quad \left[\frac{kg}{m^3} \cdot \frac{m^3}{s}\right]$$